Modeling, Analysis and Simulation of a Form-Found Civic Sculpture for Lower Manhattan

Registration Confirmation Number: 13

Keywords: Cable Net, Nonlinear Analysis, Performance Based Design, Public Sculpture, Vector Algebra.

Abstract

Analysis and design of an 8-storey cable net sculpture for a new transit center in Lower Manhattan required its designers to adopt a highly integrated computational approach. This approach combined aspects of structural analysis, form-finding, 3D and parametric modeling to achieve the artistic objectives of the sculptural form while ensuring desired performance of the extensive tensile system.

This paper presents an overview of the artistic and functional objectives of the cable net and describes in detail the design approach of the project team, including software integration, nonlinear analysis, geometric modeling, documentation and construction administration. It outlines a partial approach toward performance-based design for a system with strong aesthetic objectives and a collaborative approach to construction administration to manage risk amogst the various stakeholders.

1. INTRODUCTION

Eleven lines of the New York City subway system converge at or near the corner of Fulton Street and Broadway, one block east of the World Trade Center site and two blocks south of City Hall Park. Despite their dense tangle, these lines have evaded efficient connection for nearly a century: a legacy of disparate planning and construction practices common to the era of competitive, privatized transit operation. The eleven lines around Fulton Street, which were unified under a state agency in 1968, serve as a hub for more than 300,000 transit riders each day.



Figure 1. Project rendering of central public space, showing cable net and oculus.

In the aftermath of September 11, 2001, the Metropolitan Transportation Authority enacted plans to redevelop this hub into an efficient transfer point between nearby subway lines. They began a project to replace the labyrinthian corridors retroactively constructed to link existing lines with an efficient system of pedestrian mezzanines, concourses and underpasses to link platforms of the eleven transit lines. At the corner of Broadway and Fulton Street, the MTA planned a spacious multi-story pavilion structure to crown the new underground pedestrian network and serve as an iconic gateway to Lower Manhattan.



Figure 2. Cable net components and boundary support.

Arup has served as the prime consultant for the project, called the Fulton Street Transit Center, and has offered a wide range of multidisciplinary design services as well as overall planning and project management. Architectural design services for the transit center superstructure have been provided by Grimshaw, whose building concept included a three-storey glazed pavilion set around a centrally located, 8-storey dome structure. Topped with an inclined, circular skylight measuring 50 feet in diameter referred to as the oculus, this large central space marks the focal point of the extensive project and serves to collect and

redirect natural sunlight through the building to the exhumed sublevels below.

The central space beneath the dome and oculus offer a unique opportunity for a large-scale artistic installation, to further add character to the space and extend the architectural objective of repurposing incident sunlight to illuminate subterranean spaces. The MTA held a public art competition to find a concept for an installation.

Submitted by artist James Carpenter, the winning entry called for a metal lining offset from the interior of the dome that would provide diffuse reflection of incident sunlight into the spaces below. Through a collaborative process with the design team, the concept was refined into a steel cable net structure supporting nearly 1,000 coated aluminum infill panels using flexible, universal node connection assemblies.

2. DESIGN

The cable net extends the full height of the central public space. It is suspended from 56 connection points around the compression ring of the oculus and anchored to as many cantilevered beams at levels 2 and 3. The form of the cable net is that of a skewed hypar. Unlike a regular hypar, this form has double curvature. Moreover, the skewed form has only one axis of symmetry. As a consequence, the shape of each four-sided infill panel is unique. Its shape is set out by the lengths and intersecting angles of cable segments adjacent to each side.

The reflective infill panels are comprised of 1/8" aluminum substrate with a high performance coating applied to the interior faces. They are perforated with a regular circular pattern to control the quantity of light reflected, reduce loading on the cable net, and permit the passage of interior air currents. The panels are linked to the cable net at each corner by cruciform connectors affixed to the nodes of the net.

The strategy for securing each panel follows with conventional cladding practice. The panel is suspended from its top corner by a pin fed through a standard hole in the corresponding connector. The angle and position of the panel in space is established at the bottom corner, where a pin is fed through a vertically slotted hole that allows for correct positioning with a minimum amount of restraint. Holes in connector arms at the left and right corners are oversized to allow for independent movement of the net due to changes in temperature across the 8-storey net or air pressure caused by large air intake grills located at its midpoint.

The net is comprised of 112 stainless steel cable pairs. The cables, which are $\frac{1}{4}$ " in diameter, are mechanically swaged at each node. Swages are through-bolted with cruciform connectors arranged in between the opposing cable pairs. Stainless steel rods are used for top and bottom ties as well as ring elements.

2.1. Form-Finding

An initial visual assessment of the skewed hyperbolic form proposed by the artist indicated that a form-found tensile system could be developed to fit the desired geometry. This was enabled through the use of swages at the cable net nodes that tightly grip the cables, allowing each cable segment to carry a unique tensile force and dramatically broadening the range of geometric forms achievable using a purely tensile system.

As the desired final form was known, the "form-finding" process entailed ascertaining the relative magnitude of tensile force required in each cable segment to achieve that form. Arup developed a computational model of the desired cable net geometry, including the position of all nodes. It was first run with a uniform 900-pound tensile force in each cable pair. The resulting found form varied significantly from the initial, desired geometry due to the generalized prestress force. Where the force entered was larger than the true prestress, the cable segment shortened. Where the force was smaller, the cable segment lengthened.

These geometric shifts from the initial geometry, while undesirable in principle, result in a redistribution of the generalized initial prestress. The shortening of segments that were overstressed, for example, leads directly to a reduction in the force in the element following form finding and prestress redistribution. Put simply, the prestress redistribution inherent in the form-finding process is selfcorrecting.

These redistributed forces were again applied to the desired architectural geometry. The resulting node displacements were smaller than those observed in the first iteration, and the redistribution of prestress resulting from this second form-finding routine further approached the correct prestress values for each cable segment.

The engineers at Arup then developed a custom embedded into Microsoft Excel, which automatically ran subsequent iterations, each time applying the redistributed loads to the original geometry and performing a form-finding routine. The routine terminated once convergence indicators were met, following the 113th iteration. The resulting prestress distribution pattern exhibited maximum average values of 906 pounds at the top and bottom rows and minimum average values of 886 pounds at mid height.

2.2. Geometric Analysis

In order to achieve this level of flexibility, the design team developed a comprehensive mathematical model of the cable net, using linear algebra to represent each onedimensional linear component (e.g. cable segments, armatures, fold lines) as a unique vector in Euclidean space. This effort was carried out in Microsoft Excel with the assistance of Microsoft Visual Basic for Applications. These vector representations were then used to perform geometric checks to test the need for flexibility in the connection assemblies. Mathematical representations and sample geometric checks are illustrated in Figure 2 below.

Cartesian coordinates of nodes from the initial form finding model were used as the basis for mathematical model generation. Vector equations were developed to represent each cable segment. Four adjacent vectors can be used to define the shape of a single infill panel, as illustrated in Figure 2a. Due to the three-dimensional shape of the net, the four nodes that define a single panel are not co-planar. Each four-sided panel was thus treated as a compound shape derived from two three-sided polygons sharing a common side. That common side was defined as the "fold line" and denoted \vec{F} , a vector oriented by the left and right corner nodes of the infill panel.

Cable segment equations were then used to check the corner angles (θ) for all panels (Figure 2b). From the tabulated data, the maximum and minimum panel angles for the entire system were readily extracted. Once known, the range of panel angles was used to establish panel edge details such as corner radii and perforation patterns.



Figure 3. Mathematical model of cable net geometry.

The horizontal and vertical armatures of each connector were likewise modeled as vectors with fixed length. These vectors were set out using the location of connector bolt holes on each panel (Figure 2c). The top and right bolt holes were set out first, located a fixed distance from the nearest cable node and aligned respectively on the panel angle bisector and the fold line. As the armatures are effectively straight, vectors were drawn from these bolt holes to the nearby cable node and subsequently projected a fixed distance beyond into adjacent panels, to set out bottom and left bolt holes. This tabulated data was used to confirm that all bolt holes were suitably spaced from panel edges, and to obtain the range of angles between horizontal and vertical armatures required of a flexible universal assembly, as shown in Figure 3.



Figure 4. Perforated infill panels and universal node assemblies, with cruciform armatures and cable swages.

Figure 2d illustrates a geometric check of the angle between triangular components of a single four-sided panel. This check was necessary in order to ascertain whether planar panels could be produced, or if connector tolerances required each panel to be bent about its fold line. Analysis ultimately revealed that bending was necessary, however the range of fold angles was sufficiently slight to allow all panels to be bent to a single, averaged fold angle, thus generalizing an otherwise unique characteristic of each panel.

Numerous other geometric checks were carried out on the mathematical model of the cable net in order to fully quantify the geometric byproducts of the skewed hypar shape. Taken as a whole, these checks enabled the Arup design team to develop a range of flexible, universal assembly components that capitalized on the economies of mass production to control costs associated with the fabrication and installation of the cable net system.

2.3. Understanding Movements

The form-finding analysis performed to establish a suitable strain distribution pattern was predicated on several assumptions about the net's real-world environment. First, it assumed uniform ambient temperature and no loading from internal air pressure. Additionally, the analysis assumed an exact match between the prescribed prestress distribution and that which would be applied to the system in the field.

In reality, the 8-storey space will inevitably have thermal gradients, as warm air collects near the oculus and cool, conditioned air is diffused within the occupied space below. The ventilation strategy for the building relies on the development of air currents that must pass through the panels of the net, resulting in pressure drops across the perforated surface created by infill panels and generating loads that will influence the shape of the net.



Figure 5. Axial force in cable pairs under nominal panel loading and prestress.

A realistic take on the process of tensioning the net must involve an acknowledgment of errors inherent in the final values. A review of industry standards suggested variations in applied tension loads on the order of $\pm 20\%$.

Because of its scale, the cable net must be viewed as a dynamic structure, one that at any time assumes a unique form that corresponds to the sum total of loads applied. Those loads, which include the weight of panels, thermal strains, air pressures and applied prestress forces, will vary over time. Consequently the position of each cable net node will also vary. Put another way, each unique combination of loads yields a corresponding unique cable net shape.

It is essential to understand and quantify the maximum conceivable movements in the nodes of the cable net under all realistic environmental conditions. Were any of these movements to result in the transfer of tensile forces to the infill panels, such forces would quickly overwhelm the delicate aluminum elements. The cruciform connector arms thus hold each panel in place with minimal restraint in order to allow for unrestrained movement of adjacent cable net nodes.

These degrees of freedom are provided by the slotted and oversized holes in the connector arms on the left, right and bottom corners of each panel. The dimensions of these non-standard holes must be coordinated with the maximum conceivable movements in order to provide sufficient freedom of movement and avoid the transfer of force into the aluminum panels.

Arup engineers studied each environmental characteristic and developed various realistic scenarios to cover the range of corresponding loading conditions. To determine the magnitude of air pressures generated by interior air currents, a computation fluid dynamic model was developed in ANSYS. The program identified the pressure drop across panels given the known volumetric flow of intakes and diffusers throughout the building and the degree of permeability in each area of the cable net.

The scenarios for each environmental characteristic were then superimposed to generate 815 unique load combinations, each with a presumed unique cable net shape. A static non-linear analysis was conducted for each of the 815 load combinations. A custom subroutine was written to interrogate the position of each node under each load case and determine the largest changes to the support conditions of each panel. An increase in the distance between left and right corner supports was referred to as a "panel width change"; the peak value implied the required dimensions of the oversized holes at these support points. Correspondingly, an increase in the distance between top and bottom corner supports was referred to as a "panel height change", which provided information on the necessary dimensions of the vertically slotted hole at the bottom connection point.

Analysis revealed that the largest resolved movement of any node under any perceivable load combination is approximately 2 1/16". Broken into its components however, this maximum movement results in a change of only 0.23" in panel width and 0.08" in panel height. The dimensions of oversized and slotted holes were correspondingly set to accommodate these movements in addition to acceptable construction tolerances of 0.25" in either direction.



Contours of Static Pressure (lb/ft2)

Figure 6. A computational fluid dynamic (CFD) model was generated to determine air pressure incident on each panel.

2.4. Modeling a Universal Connector

In conjunction with the project architects, Arup developed a universal cruciform connector to provide adequate panel support in the variety of configurations that result from the skewed hypar shape. Each connector is comprised of a horizontal and vertical armature, linked by a bolt at their midpoints and set between swaged cable pairs. All elements are free to rotate about the bolt axis.

This rotational freedom is needed to allow the connector armatures to conform to the various required configurations. The armatures, which are nominally 1/4" in thickness, taper to 1/8" at their midpoints to minimize the effective thickness of the armature assembly, and the corresponding eccentricity between opposing cable pairs.

The process of designing the universal connector armature required the selection of geometry that incorporated the space-saving taper while facilitating the free rotation of armature elements to accommodate the various panel configurations.

Arup developed a parametric model in Digital Project to assess the performance of various connector armature designs. Each connector component, including the armatures, bolt and swages, were individually modeled as discrete parts and subsequently assembled into a single component. That assembly was instantiated into a global model containing the 896 cruciform connector nodes.

Set-out instruction intrinsic to each connector instance facilitated the rapid configuration of each given the position of adjacent nodes in all directions. A separate clash detection program was then used on the solids to confirm the rotational adequacy of the universal part design.

Limitations to computing power available at the time led to instantiation routines that took days to complete. The design team observed that the instantiation of solid components to be by far the most time consuming; points and lines that established the location and orientation of the components, on the other hand, were much faster to instantiate.



Figure 7. Parametric definition of horizontal armature.



Figure 8. Custom routines instantiate the connector prototype throughout the node cloud.

In order to accelerate the process of studying each armature design, solid components were removed and only set-out lines were instantiated. A custom script was written to interrogate the relative angles between each armature pair, and the most severe angles were then modeled using the solid components to test their suitability. This modified approach to parametric modeling proved exceptionally fast, and instantiation of armatures on all 896 points was completed in a matter of seconds. The geometry of the armatures for the universal connector was optimized for visual and performance criteria through the parametric model, and the final design was extracted from the individual part models and presented as simple 2-dimensional sections and elevations in contract drawings.

3. DOCUMENTATION AND CONSTRUCTION

The design team employed a hybrid approach to documentation, largely embracing prescriptive design but thoughtfully integrating performance criteria to ensure the selection of a specialty subcontractor knowledgeable and experienced in the construction of such a system. In addition, Arup was motivated to bring in performance requirements in order to facilitate a productive dialog with the contractor that would allow the designer to more effectively monitor and observe the contractor's coordination, shop drawing preparation, fabrication and assembly of the cable net. The resulting working environment promoted open dialog on potential construction issues and a balanced approach toward project risk management.

Construction of the transit center superstructure and fitout of the area bounded by its enclosure and foundation beneath were packaged into a single construction contract that was awarded to a joint venture comprised of Plaza and Schiavone (PSJV) in August of 2010. The complex and unconventional nature of the cable net compelled the general contractor to bring on several specialist subcontractors to fabricate and install the tensile structure. Coordination of the cable net was managed by Enclos, the subcontractor also selected to provide facade components for the pavilion enclosure and the oculus. Cables and custom connection elements, such as the cruciform armatures, were provided by TriPyramid Structures, Inc. The anodized aluminum infill panels were provided by Durlum out of Schopfheim, Germany. Along with the steel fabrication and erection subcontractor STS Steel, these members of the construction team exhibited skill and expertise that added to the value of the design through open collaboration. Performance-based provisions in the contract documents, which were intended to incite discourse among members of the design and construction teams, brought about fruitful if occasionally intense discussion over elements of the design and strategies for its execution.

Fabrication of the cable net cables, connectors and panels are currently underway by suppliers and subcontractors. Installation of the tensile structure, which has an estimated construction cost of \$3.8M, is scheduled to begin in February of 2012.

4. CONCLUSION

The Fulton Street Transit Center cable net is a powerful example of the capacity of a large tensile structure to define a landmark public space. A comprehensive understanding of the system geometry across the form, assembly and subcomponent levels was necessary in order to ensure that cost-effective, universal components embodied the necessary flexibility to accommodate the complete range of aconnection arrangements and patterns. This understanding was achieved through the development of a complete mathematical model of the system to facilitate rapid geometric checks throughout the component detailing phase, and a live parametric model of universal components to ensure a final design informed by system needs across a broad range of design disciplines.

Acknowledgements

The authors would like to thank Ricardo Pittella, P.E., Associate Principal, John Batchelor, Director, Craig Covil, and Markus Schulte, Principals, at Arup for their supervision and guidance during the entire project. We would also like to thank James Carpenter, Richard Kress, Cameron Wu, and Torsten Schlauersbach at JCDA and Grimshaw Architects, PC, New York for their collaboration on the project. Special thanks are extended to the MTA, in particular Uday Durg, Raj Desai and Virginia Borkoski. All images courtesy of MTACC/NYCT and Arup. All images © MTACC/NYCT.

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